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RESIDENCE-TIME DISTRIBUTION OF PARTICLES IN THE POST OF A CIRCULATING BOILING BED

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The residence-time distributions of particles in the lifting post of a circulating boiling bed were calculated on the basis of a model of longitudinal mixing of particles in this bed. It is shown that this model is well approximated by the standard γ-distribution. The dependence of the residence-time distribution function on the main parameters of the circulating boiling bed has been determined. The calculated and experimental distribution functions were compared.

Introduction. Reactors with a boiling bed or a circulating boiling bed, operating in the uninterrupted-cycle regime, have, along with unquestionable advantages, a common drawback — different particles are characterized by different times of residence in the active zone of such a reactor [1, 2]. The difference between the residence times of solid-material particles can be reduced by organization of their recirculation; however, the residence-time distribution of particles in a circulating boiling bed remains inhomogeneous.

The aim of the present work is to determine the residence-time distribution function (RTDF) of particles in the post of a circulating boiling-bed on the basis of the model of longitudinal mixing of particles in a circulating boiling bed, formulated in [3], and the dependence of this function on the characteristics of the system, including the height of the near-bottom zone of a boiling bed, in which a practically ideal mixing of particles is realized.

The RTDF is determined as [1]

$$E(t) = \frac{\rho(H)\, wS}{m_0}\, c(t, H)\,, \tag{1}$$

where m_0 is the amount of a labeled material introduced into the lower zone of a circulating boiling bed. At $J_s = \rho(H)w$, dependence (1) takes the form

$$E(t) = \frac{J_s S}{m_0}\, c(t, H)\,. \tag{2}$$

The quantities m_0 and $c(t, H)$ should be determined with allowance for the conditions under which the labeled material is introduced into the lower part of the circulating boiling layer.

Instantaneous Introduction of a Labeled Material. This ideal case is most suitable for investigating the function $E(t)$. The value of m_0 is determined from the formula

$$m_0 = \rho_{fb} c_0 H_{fb} S\,. \tag{3}$$

Then, from (2), we obtain

$$E(t) = \frac{J_s}{\rho_{fb} H_{fb} c_0}\, c(t, H)\,. \tag{4}$$

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Expression (4) relates the desired function $E(t)$ to the output concentration of the labeled particles $c(t, H)$ and the parameters of a circulating boiling bed and its near-bottom zone (J_s, H_{fb}, ρ_{fb}). The value of a circulating particle mass flow J_s is an independent parameter, and H_{fb} and ρ_{fb} are determined as [4]

$$H_{fb}/H = 1.25\text{Fr}_t^{-0.8} J_s^{-1.1} \,, \tag{5}$$

$$\rho_{fb}/\rho_s = 0.33\text{Fr}_t^{-0.045} \,. \tag{6}$$

Taking into account (5) and (6), we obtain the following expression for $E(t)$:

$$E\,(t) = 2.4\text{Fr}_t^{0.85} \bar{J}_s^{-0.1} \frac{c\,(t, H)}{c_0} \frac{u - u_t}{H} \,, \tag{7}$$

and, for the dimensionless RTDF,

$$E\,(t) = E\,(t)\,H/(u - u_t) = 2.4\text{Fr}_t^{0.85} \bar{J}_s^{-0.1} \frac{c\,(t, H)}{c_0} \,. \tag{8}$$

The function $c(t, H)$ is determined by solving the problem on the longitudinal mixing of particles under corresponding boundary conditions. It has been shown in [3] that a mixing process is defined by the system of equations

$$A\rho_1 \frac{\partial c_1}{\partial t} + A\rho_1 u_1 \frac{\partial c_1}{\partial x} = \beta_* \,\rho\,(c_2 - c_1) \,, \tag{9}$$

$$B\rho_2 \frac{\partial c_2}{\partial t} - B\rho_2 u_2 \frac{\partial c_2}{\partial x} = (\beta_* \,\rho + A\rho_1 \,\beta_1)\,(c_1 - c_2) \,. \tag{10}$$

The following boundary conditions define the instantaneous introduction of the labeled material:

$$c_1\,(0, x) = c_2\,(0, x) = 0 \,; \quad c_1\,(0, H_{fb}) = c_0 \,;$$

$$x = H \,, \quad c_1 = c_2 \,; \tag{11}$$

$$x = H_{fb} \,, \quad \rho_{fb} H_{fb} \frac{\partial c_1}{\partial t} + A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2 = 0 \,.$$

Let us write system (9)–(11) in the dimensionless form [3]:

$$\frac{\partial c_1}{\partial t'} + u_1' \frac{\partial c_1}{\partial x'} = \frac{1}{\text{Pe}} \frac{u_1' + u_2'}{u_2' + (x')^{0.82}} \,(c_2 - c_1) \,, \tag{12}$$

$$\frac{\partial c_2}{\partial t'} - u_2' \frac{\partial c_2}{\partial x'} = \frac{1}{\overline{\text{Pe}}} \frac{u_1' + u_2'}{u_1' + (x')^{0.82}} \,(c_1 - c_2) \,, \tag{13}$$

$$c_1\,(0, x') = c_2\,(0, x') = 0 \,; \quad c_1\,(0, H_{fb}') = c_0 \,;$$

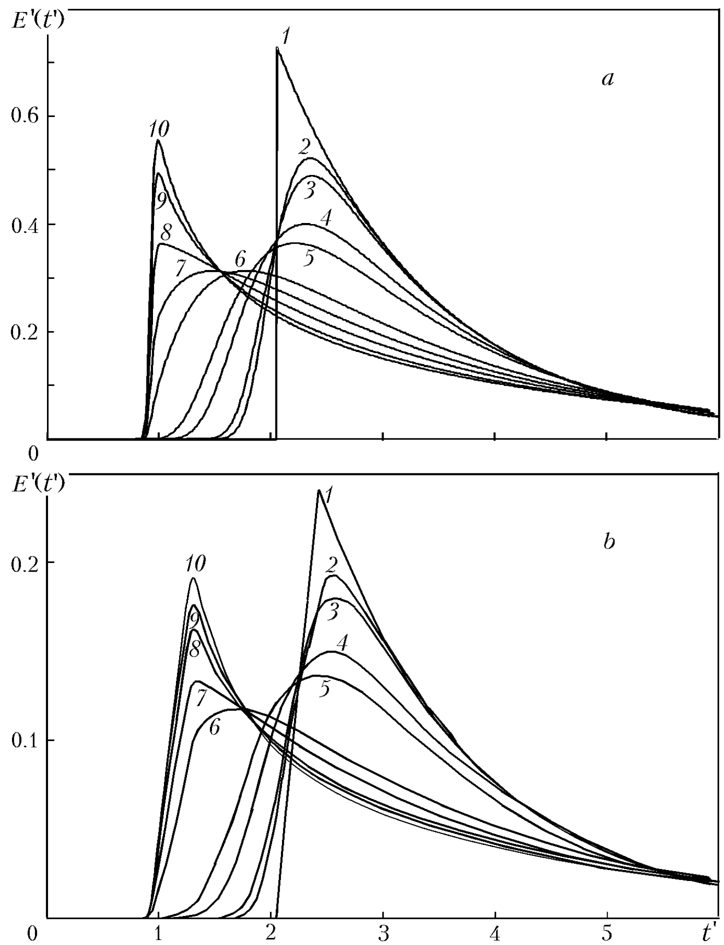

Fig. 1. Dimensionless RTDF obtained for different Pe numbers: a) instantaneous introduction of a labeled material; b) introduction of a labeled material for a finite time internal Δt = 0.4 sec (Pe = 0 (1), 0.005 (2), 0.01_(3), 0.05 (4), 0.1 (5), 1 (6), 2 (7), 5 (8), 10 (9), and 100 (10)). $\mathrm{Fr_t}$ = 0.156, $\bar{J}_\mathrm{s}$ = 0.02, c_0 = 1.

$$x' = 1\,, \quad c_1 = c_2\,; \tag{14}$$

$$x' = H'_{\mathrm{fb}}\,, \quad mH'_{\mathrm{fb}}\frac{\partial c_1}{\partial t'} + \frac{u'_2 + (H'_0)^{0.82}}{u'_1 + u'_2}\,u'_1 c_1 - \frac{u'_1 - (H'_{\mathrm{fb}})^{0.82}}{u'_1 + u'_2}\,u'_2 c_2 = 0\,,$$

here, the quantities u'_1, u'_2, and m are calculated by the formulas [3]

$$u'_1 = 1\,, \quad u'_2 = 0.1\mathrm{Fr_t^{-0.7}}\,, \quad m = 0.4\mathrm{Fr_t^{-0.7}}\,\bar{J}_\mathrm{s}^{-0.1}\,. \tag{15}$$

Taking into account (5) and (15), we will obtain the following critical equation for the residence-time distribution of particles (8):

$$E'(t') = \varphi\,(\mathrm{Fr_t}, \bar{J}_\mathrm{s}, \mathrm{Pe}, t')\,, \tag{16}$$

from which it is evident that the dimensionless RTDF is determined by the three dimensionless quantities $\mathrm{Fr_t}$, $\bar{J}_\mathrm{s}$, and Pe.

TABLE 1. Parameters of the γ-Distributions

Pe	Λ	α	β	τ	Δ, %
0	1.19	0.91	0.47	2.05	2.14
0.005	0.92	1.88	1.37	1.79	1.56
0.01	0.93	2.00	1.35	1.70	1.37
0.05	0.94	2.24	1.21	1.36	0.90
0.1	0.94	2.21	1.10	1.18	0.75
0.5	0.95	1.68	0.75	0.91	0.46
1	0.95	1.36	0.60	0.91	0.38
2	0.92	1.18	0.57	0.91	1.04
5	0.86	1.10	0.64	0.91	2.09
10	0.83	1.08	0.70	0.91	2.62

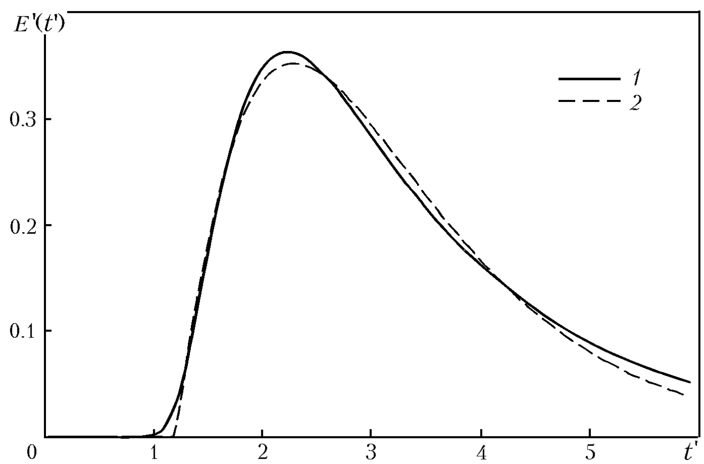

Fig. 2. Comparison of the RTDFs calculated by 1) (12)–(14) and 2) (20).

The boundary problem (12)–(14) was solved numerically by the method of finite differences. Figure 1a shows the change in the RTDF with change in the Peclet number Pe. In the case where Pe = 0, $c(t', 1)$ was calculated by the formula

$$c = c_0 \exp\left(-2.4\mathrm{Fr}_{\mathrm{t}}^{0.85}, \bar{J}_{\mathrm{s}}^{-0.1} (t' - t'_{\mathrm{d}})\right), \tag{17}$$

which represents, when the delay time t_{d} is taken into account, a solution of the equation

$$\rho_{\mathrm{fb}} H_{\mathrm{fb}} \frac{dc}{dt} + J_{\mathrm{s}} c = 0\,. \tag{18}$$

Note that (18) follows from the boundary condition (11) at the point $x = H_{\mathrm{fb}}$ at $c_1 = c_2 = \mathrm{c}$. The dimensionless delay time is determined by the dependence [3]

$$t'_{\mathrm{d}} = 5.5\left(1 - (H'_{\mathrm{fb}})^{0.18}\right). \tag{19}$$

For analytical representation of the family of curves shown in Fig. 1a, we used the standard γ-distribution [5]:

$$E'(t') = \Lambda \frac{\beta^{\alpha}}{\Gamma(\alpha)} (t')^{\alpha-1} \exp(-\beta (t' - \tau))\,. \tag{20}$$

The parameters Λ, α, β, and τ, determined for different values of the Pe number by the method of least squares, are presented in Table 1. In Fig. 2, the approximation function $E'(t')$, calculated by (20), is compared with the function obtained as a result of the numerical solution of (12)–(14).

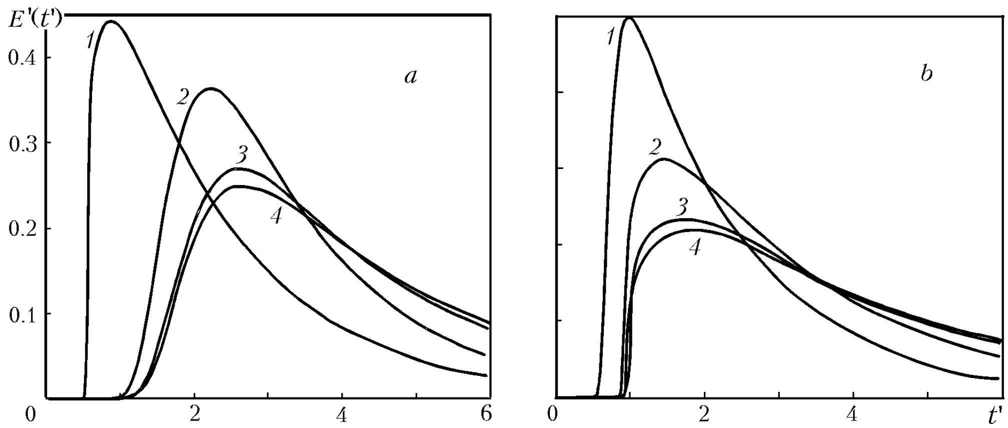

Fig. 3. Dimensionless RTDFs obtained for different values of $\overline{J}_s$ at $c_0 = 1$, $Fr_t = 0.156$, and Pe = 0.1 (a) and Pe = 1 (b): $\overline{J}_s = 0.1$ and $H_{fb}/H = 0.44$ (1); 0.02 and 0.074 (2); 0.002 and 0.0059 (3); 0.0002 and 0.00047 (4).

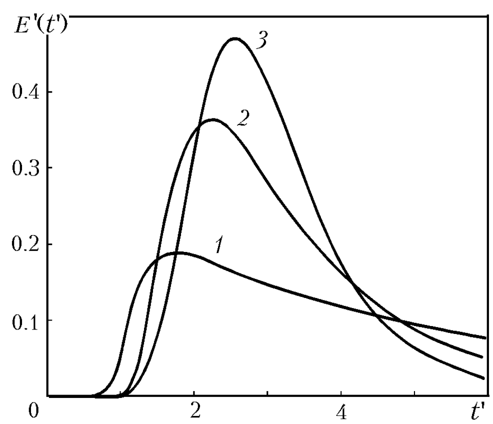

Fig. 4. Dimensionless RTDFs obtained for different values of Fr_t at $c_0 = 1$, Pe = 0.1, and $\overline{J}_s = 0.02$: $Fr_t = 0.04$ (1), 0.156 (2), and 0.6 (3).

Figures 3 and 4 show the evolution of the RTDF with change in the quantities Fr_t and $\overline{J}_s$. Of special interest are the dependences $E'(t')$ obtained for different values of $\overline{J}_s$, which are presented in Fig. 3a and b. As follows from (5), the quantity $\overline{J}_s$ determines the height of the near-bottom boiling layer at a constant value of Fr_t. As has already been intimated, in this region there arises an ideal mixing of particles characterized by the exponential dependence $E'(t')$ [6]. Because of this, with increase in $\overline{J}_s$, the RTDF approaches, in shape, an exponential function.

Introduction of a Labeled Material for a Definite Time. This case corresponds to the actual experimental conditions under which the RTDF is determined. It is assumed that a labeled material is introduced with a constant mass rate j into the near-bottom layer of a circulating boiling bed for the time Δt. In this case, the value of m_0 in (2) is equal to

$$m_0 = j\Delta tS\,. \tag{21}$$

It follows from (2) that

$$E\,(t) = \frac{J_s}{j\Delta t}\,c\,(t, H)\,, \tag{22}$$

and, for the RTDF in dimensionless form, we obtain

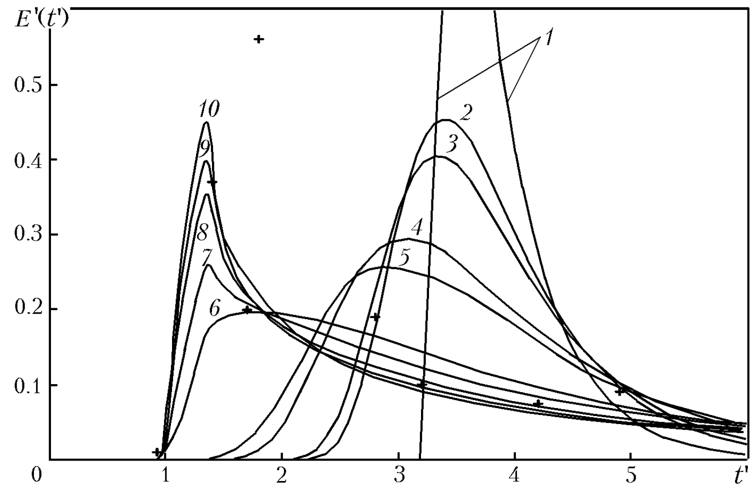

Fig. 5. Comparison of the experimental RTDF (points)_[7] with the RTDFs calculated by (9), (10), and (24)–(26) at $\mathrm{Fr_t}$ = 0.45 and J_s = 0.0057. Designations 1–10 are identical to those in Fig. 1.

$$E'(t) = E(t)\,\Delta t = \frac{J_s}{j}\,c\,(t, H)\,. \tag{23}$$

The value of $c(t, H)$ was determined using Eqs. (9) and (10) with boundary conditions differing from the boundary conditions (11):

$$c_1(0, x) = c_2(0, x) = 0\,; \quad x = H\,, \quad c_1 = c_2\,; \quad x = H_{\mathrm{fb}}\,;$$

$$\text{a) } t \le \Delta t: \quad \rho_{\mathrm{fb}} H_{\mathrm{fb}} \frac{\partial c_1}{\partial t} + A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2 = j\,; \tag{24}$$

$$\text{b) } t > \Delta t\,; \quad \rho_{\mathrm{fb}} H_{\mathrm{fb}} \frac{\partial c_1}{\partial t} + A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2 = 0\,.$$

Figure 1b presents the results of calculating $E'(t')$ by (23) at $J_s = j$ and Δt = 0.4 sec. In the case where Pe = 0, $c(t', 1)$ was determined as

$$\text{a) } t_d \le t \le t_d + \Delta t: \quad c(t', 1) = 1 - \exp\left(-2.4\mathrm{Fr_t^{0.85}}\,\bar{J}_s^{-0.1}\,(t' - t'_d)\right); \tag{25}$$

$$\text{b) } t > t_d + \Delta t: \quad c(t', 1) = c_0 \exp\left(-2.4\mathrm{Fr_t^{0.85}}\,\bar{J}_s^{-0.1}\,(t' - t'_d - \Delta t')\right), \tag{26}$$

where $c_0 = 1 - \exp(-2.4\mathrm{Fr_t^{0.85}}\,\bar{J}_s^{-0.1}\Delta t')$. The quantity t'_d was determined from (19). Formula (25) is a solution of the equation

$$\rho_{\mathrm{fb}} H_{\mathrm{fb}} \frac{dc}{dt} + J_s\,c = j\,, \tag{27}$$

following from the boundary condition a) in (24) at $c_1 = c_2 = 0$ and $c(t_d) = 0$. Note that expression (26) is identical to (17).

Figure 5 shows the calculated RTDFs compared with the experimental ones obtained in [7]. It is seen that the calculated and experimental functions agree best at large Pe numbers (10–100), which corresponds to the values of the transfer coefficient $\beta_* \approx 0.01–0.1$ 1/sec. They correlate well with the value of $\beta_* = 0.07$ 1/sec obtained in [3] as a result of the processing of experiential data on mixing of particles.

Conclusions. It has been shown that the model of longitudinal mixing of particles in a circulating boiling bed proposed in [3] allows one to determine the RTDF and the influence of the main dimensionless parameters $\mathrm{Fr_t}$, $\overline{J}_s$, and Pe on it. The RTDFs obtained are well approximated by the standard γ-distribution. The influence of the near-bottom boiling layer on the RTDF has been determined (Fig. 3). Comparison of the calculated and experimental RTDFs (Fig. 5) allowed us to estimate the coefficient of transfer of particles between the core of a circulating boiling bed and its ring zone: $\beta_* = 0.01–0.1$ 1/sec. These values correlate well with the value of β_* determined earlier.

NOTATION

A, part of the horizontal cross section of the post of a circulating boiling bed occupied by rising particles (core of the bed); B, part of the horizontal cross section of the post occupied by particles moving down (ring zone); c_1 and c_2, dimensionless concentrations of labeled particles in the core of the bed and in its ring zone; c_0, initial dimensionless concentration of labeled particles in the near-bottom boiling bed; $E''(t') = E'(t')/(1 - \exp(-2.4\mathrm{Fr_t^{0.85}}\,\overline{J}_s^{-0.1}\Delta t'))$; $\mathrm{Fr_t} = (u - u_t)/gH$, Froude number; g, free fall acceleration, m/sec^2; H, height of the post; H_{fb}, height of the near-bottom boiling layer, m; $H'_{fb} = H_{fb}/H$, J_s, circulating mass flow, kg/(m^2·sec); $\overline{J}_s = J_s/\rho_s(u - u_t)$; $\mathrm{Pe} = (u - u_t)/\beta_* H$, $\overline{\mathrm{Pe}} = \mathrm{Pe}\Big/\left(1 + 0.82\dfrac{u'_1 u'_2}{u'_1 + u'_2}\mathrm{Pe}\dfrac{1}{x'}\right)$, Peclet number; S, cross-section area of the post, m^2; t, time, sec; $t' = t(u - u_t)/H$, dimensionless time; u, rate of gas filtration, m/sec; u_t, velocity of travel of an individual particle, m/sec; $u'_1 = u_1/(u - u_t)$, $u'_2 = u_2/(u - u_t)$, dimensionless velocities of particles; $w = J_s/\rho(H)$, transport velocity of a particle at the output of the post, m/sec; x, vertical coordinate, m; $x' = x/H$; β_*, coefficient of transfer of labeled particles, 1/sec; $\beta_1 = -\dfrac{u_1}{A\rho_1}\dfrac{dA\rho_1}{dx}$, coefficient accounting for the directed flow of particles from the core of the bed to the ring zone, 1/sec; $\Gamma(\alpha)$, gamma function; ρ_1 and ρ_2, density of the bed at the core and in the ring zone, kg/m^3; $\rho = A\rho_1 + B\rho_2$, average density of the bed, kg/m^3. Subscripts: 1, core of the bed; 2, ring zone; d, delay; fb, near-bottom boiling layer; s, particles; t, condition of travel of an individual particle.

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